RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2014

SECOND YEAR

Date : 27/05/2014   **MATHEMATICS (General)**

Time : 11 am – 2 pm   Paper : IV   Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

(*Answer **any four** questions*)

1. a) Apply μ-test to examine the convergence of $\int_1^{\infty} \frac{x}{(1+x)^3} \, dx$.   [3]

   b) Express $\int_0^{\frac{\pi}{2}} \sin^4 \theta \cos^6 \theta \, d\theta$ as a Beta function.   [2]

2. a) Find the value of $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$.   [4]

   b) Obtain the Cauchy principal value of $\int_{-1}^1 \frac{1}{x^3} \, dx$.   [1]

3. a) Evaluate $\int_0^{\infty} 5^{-x^2} \, dx$.   [3]

   b) Test the convergence of $\int_1^2 \frac{\sqrt{x}}{\log x} \, dx$.   [2]

4. a) Prove that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} \, dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} \, dx$.   [3]

   b) Evaluate $\int_0^{2\pi} \int_{a\sin\theta}^a r \, dr \, d\theta$.   [2]

5. a) Find the total length of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.   [3]

   b) Find the area of the surface of revolution of $y = x^2$ between $x = 0$ and $x = \sqrt{2}$ about y-axis.   [2]

6. Find the volume of revolution of the area cut off from the parabola $9y = 4(9-x^2)$ by the line 4x+3y=12 about x-axis.   [5]

Group – B

(*Answer **any two** questions*)

7. Find the general solution of the differential equation $\frac{d^2 y}{dx^2} + 9y = \sec 3x$.   [5]

8. Solve the differential equation $(x+1)^2 \frac{d^2 y}{dx^2} - 4(x+1) \frac{dy}{dx} + 4y = x^2$.   [5]

9. Show that the orthogonal trajectories of the system of curves $x^{\frac{4}{3}} + y^{\frac{4}{3}} = a^{\frac{4}{3}}$ is $x^{\frac{2}{3}} - y^{\frac{2}{3}} = c^{\frac{2}{3}}$.   [5]

Group – C

Unit – I

(*Answer **any four** questions*)

10. a) State the Axioms of Mathematical Probability.
   b) Derive classical definition of probability from axioms of mathematical probability.   [2+3]

11. a) Explain the term "Mutually Exclusive events" with example.
    b) Find the probability that a leap year selected at random will contain 53 Sundays. [2+3]

12. a) State the conditional probability of an event A on the hypothesis that another event C has occured.
    b) Two cards are drawn successively from a pack without replacing the first (i) If the first card is a spade, find the probability that the second card is also a spade. Also find the probability(ii) that both cards are spades. [1+4]

13. a) State Baye's theorem on conditional probabilities.
    b) Three cards are drawn at random from a pack of 52 cards. Find the probability of getting— (i) two aces, (ii) two king cards. [2+3]

14. a) Consider the random experiment of throwing a coin. We may define X(head) = 1 and X(head) = 2 and X(tail) = 2. Is X is a random variable? If not, give reason?

    b) A random variable X can assumes values –2, 0, 3 with probabilities $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$ respectively. Determine the distribution function. [2+3]

15. a) For what value of K will the function f(x) = kx, x = 1,2, ...n be the probability density function of a continuous random variable.
    b) From an urn containing 2 white and 3 black balls, balls are successively drawn without replacement. Find the mathematical expectation of the number of black balls preceding the first white ball. [2+3]

Unit – II
(*Answer **any five** questions*)

16. Draw a Histogram to represent the following frequency distribution [5]

Marks	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
No. of Students	7	6	8	9	4	4	3

17. The median and mode of the following frequency distribution are respectively known as 27 and 26. Find the values of a and b : [5]

Value	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	a	20	12	b

18. Find out the skewness of the series by the method of moments : [5]

Measurement :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	1	3	4	2

19. Write down the density function of χ^2-distribution. State clearly Type-I and Type-II error. [1+4]

20. a) If the regression co-efficients are $b_{yx} = -\frac{3}{5}$ and $b_{xy} = -\frac{3}{20}$, find the value of correlation coefficients σ_{xy}. [2]

    b) The A.M of a distribution is 5, the 2nd and 3rd moments about its mean are 20 and 140 respectively, find the 3rd moments about 10 of the distribution. [2+3]

21. Calculate the mean and variance of first 'n' natural numbers. [5]

22. Workout the trend value by 4-yearly moving average from the following data on production of rice (in Kg). [5]

Year	2000	'01	'02	'03	'04	'05	'06	'07	'08	'09	'10
Production (kg)	506	620	1036	673	588	696	1116	738	663	773	1189

23. Using the data given below calculate price index number for 1990 with 1984 as base by Fisher's formula : [5]

Commodity	Price (Rs)		Quantity (tones)	
	1990	1984	1990	1984
A	9·4	4·6	90	100
B	6·5	3·8	20	25
C	5·2	2·6	10	11
D	4·2	2·3	3	4

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